SCATTERING FROM AN ARBITRARILY-LOCATED
OFF-AXIS INHOMOGENEITY IN A STEP-INDEX OPTICAL FIBER

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ABSTRACT

A Green's function formulation of an arbitrarily oriented off-axis dipole radiating in a dielectric rod waveguide is carried out. The analysis is applied to the problem of scattering from an arbitrarily-located inhomogeneity in a step-index optical fiber.

SUMMARY

There is much current interest in dielectric surface waveguides in view of their applications in fiber optical communication systems. Many theoretical and practical problems are involved. An important problem is the excitation of surface waves and radiation by infinitesimal dipole sources in cylindrical dielectric waveguides. One of the simplest structures, which is, nevertheless, of great practical utility is the cylindrical dielectric rod. Duncan[1] (1959) and Brown and Stachera[2] (1962) have studied the excitation of the circularly symmetric surface waves on a dielectric rod by a magnetic current ring, while Yip[3] (1970) investigated the excitation of the HE_{11} mode by a transversely-oriented infinitesimal dipole on the axis of a rod.

The treatment of an arbitrarily-oriented off-axis dipole involving a Green's function formulation has not been available so far. The surface-wave fields, however, can be evaluated by employing the Lorentz reciprocity theorem involving the use of the orthogonality properties of the modal fields[4] (1960). But this method does not provide any information about the radiation fields. It is the purpose of this paper to present a rigorous Green's function analysis of an off-axis and arbitrarily oriented dipole in a two-layer cylindrical dielectric waveguide. Further, the practical significance of such a formulation is illustrated by its application to the problem of radiation and mode conversion due to scattering from an arbitrarily-located discrete inhomogeneity in a step-index optical fiber.

The dielectric rod, characterized by a permittivity of $\varepsilon_1 = \varepsilon_0\varepsilon_{r1}$ and a permeability of $\mu_1 = \mu_0\mu_{r1}$ is assumed to be lossless and infinitely long with its axis coinciding with the z-axis of a cylindrical coordinate system (ρ,ϕ,z). It has a radius ρ_1 and is surrounded by an infinite cladding medium ($\varepsilon_2 = \varepsilon_0\varepsilon_{r2}$, $\mu_2 = \mu_0\mu_{r2}$). A point electric dipole with an arbitrary orientation is placed at ρ_0, ϕ_0, z_0 as shown in Fig. 1a, b. The time variation is assumed to be of the form $\exp(-j\omega t)$. Without loss of generality, z_0 is set to zero. The current density on the dipole can be expressed as

$$\vec{J}_1 = (J_\rho\hat{\rho}_0 + J_\phi\hat{\phi}_0 + J_z\hat{z}_0)\ \delta(\vec{\rho}-\vec{\rho}_0) \qquad (1)$$

where $\hat{\rho}_0$, $\hat{\phi}_0$ and $\hat{z}_0$ are unit vectors at ρ_0, ϕ_0, 0, along the radial, azimuthal and axial directions respectively as indicated in Fig. 1b and $\delta(\vec{\rho}-\vec{\rho}_0)$ is the three dimensional delta function. The evaluation of

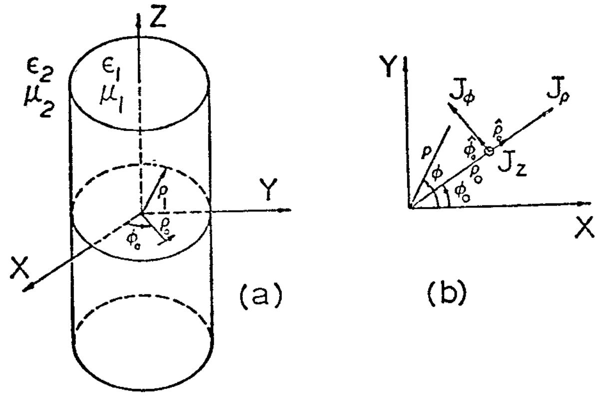

Fig. 1: Geometry of the problem.

(a) The dielectric rod and the point dipole.

(b) Current components in the dipole.

radiation and surface-wave fields amounts to solving a set of two inhomogeneous wave equations. The technique used for solving these wave equations involves: (a) a coordinate transformation in the z=0 plane such that the new origin is at ρ_0, ϕ_0, 0 and the new x axis makes an angle ϕ_0 with the old x axis, (b) transformation of the fields and the current density by means of a Fourier transform integral. After solving the wave equations for the transformed fields in the new coordinate system, we make use of the Graf's formula[5] to express the fields in terms of the old coordinates. Finally, the actual fields can be derived from

$$f(\rho,\phi,z) = \frac{k_0}{2\pi}\int_{-\infty}^{+\infty} F(\rho,\phi,\bar{\beta})\ e^{jk_0\bar{\beta}z}\ d\bar{\beta} \qquad (2)$$

$\bar{\beta}$ being the normalized axial propagation constant. In (2) f and F represent a component of the electric or magnetic field, or current in the z and $\bar{\beta}$ domains respectively. The integral in (2) is evaluated by means of a contour integration. The contribution from the branch-cut integral gives rise to the radiation fields, which is evaluated by the saddle-point method of integration. In addition to the branch-cut integral, contributions to the contour integral also come from the residues at the enclosed poles, which account for the guided surface-wave modes. The field solutions thus obtained can be immediately applied to the problem of radiation and mode conversion due to scattering from an arbitrarily-located off-axis inhomogeneity in a step-index fiber, if the current in the dipole is replaced by the current induced by the incident mode in the dielectric inhomogeneity given by $\vec{J}=-j\omega\varepsilon_0\Delta\varepsilon_r\vec{E}^i$, where $\vec{E}^i$ is the field of the incident mode and $\Delta\varepsilon_r$ is the relative permittivity difference between the inhomogeneity and its surrounding medium. The normalized total radiation power can be expressed as $\overline{P}_r = P_r/(\Delta\varepsilon_r\overline{\Delta v})^2 P_i$, where P_i is the incident modal power and $\overline{\Delta v} = \Delta v/\rho_1^3$ is the normalized volume of the inhomogeneity. The variation of $\overline{P}_r$ with the normalized frequency $V = k_0\rho_1\sqrt{\varepsilon_{r1} - \varepsilon_{r2}}$ for several values of ρ_0/ρ_1 are shown in Fig. 2.

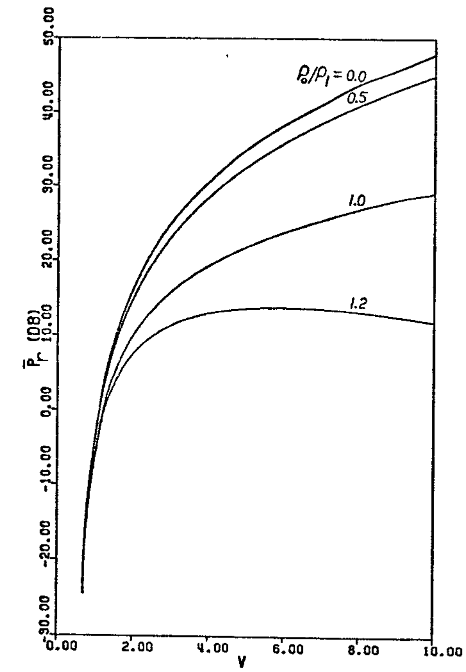

Fig. 2: The normalized total radiated power $\overline{P}_r$ versus the normalized frequency V. $\varepsilon_{r1} = 2.341, \varepsilon_{r2} = 2.250$.

For simplicity of presentation, only fiber operating in the dominant HE_{11} mode is considered. The angular position of the inhomogeneity, ϕ_0, is taken to be zero. The behaviour of the radiated power is largely determined by the variation in the incident field strength. Thus, as frequency increases, the power for $\rho_0/\rho_1 < 1$ increases, whereas for $\rho_0/\rho_1 > 1$, after reaching a maximum, decreases. This can be explained by the fact that as the frequency increases, the incident fields and hence the power become more and more confined to the core, but decay exponentially in the outer cladding.

The field solutions for the guided modes can again be directly applied to evaluate the guided field and power scattered by the inhomogeneity discussed before. Again, the normalized power scattered into a particular mode, ie., mode conversion, is given by $\overline{P}_s=P_s/(\Delta\varepsilon_r\overline{\Delta v})^2 P_i$. Figs. 3 and 4 show the variation of the normalized scattered power for several lower order modes versus frequency. In Fig. 3a, the variation of power for the scattered HE_{11} mode is shown. For values of $\rho_0/\rho_1 < 1$

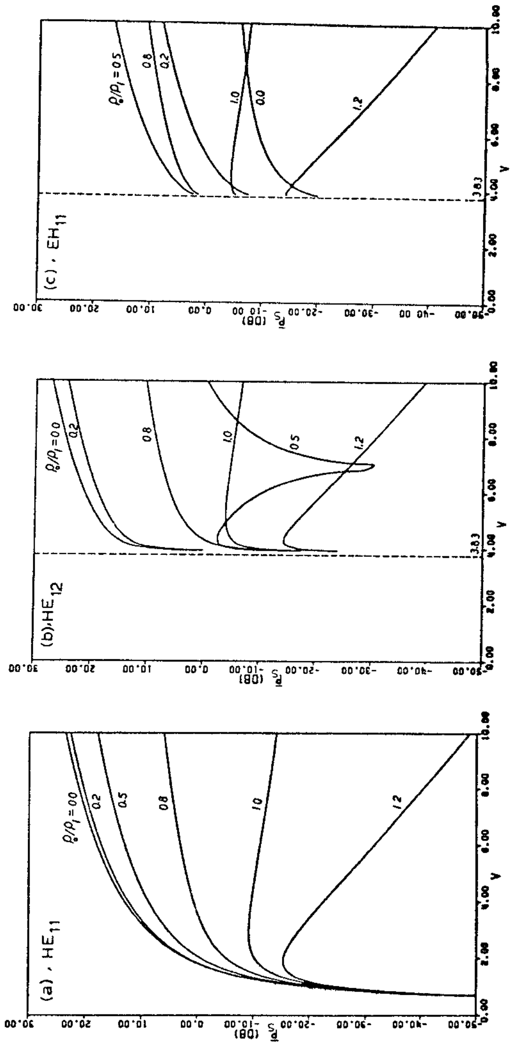

Fig. 3: The normalized scattered modal power $\overline{P}_s$ versus the normalized frequency V. $\varepsilon_{r1} = 2.341, \varepsilon_{r2} = 2.250$. (a) HE_{11} (b) HE_{12} (c) EH_{11}

the power increases with frequency, whereas for $\rho_0/\rho_1 > 1$ it decreases rapidly after reaching a maximum. This can be explained by the same reason advanced for the radiation power. Variations of power for the scattered HE_{12} and EH_{11} modes are shown in Fig. 3b,c and for the TM_{01}, HE_{21} and HE_{31} modes in Fig. 4a,b,c respectively. It is readily observed that they all have the general behaviour as the HE_{11} mode. The HE_{12} mode at $\rho_0/\rho_1 = 0.5$, however, exhibits a peculiar behaviour. After an initial increase, the power of this mode drops to -30.1 dB at V=7 and then rises to -0.38 dB at V=10. This drop in the power is caused by the fact that the field strength of this mode for certain values of ρ_0, ϕ_0 and V becomes very small.

The Green's function solution of the problem treated is exact and can be used to check the accuracy of the approximate solutions. Snyder[6] (1969) has used the infinite medium approximation for evaluating the

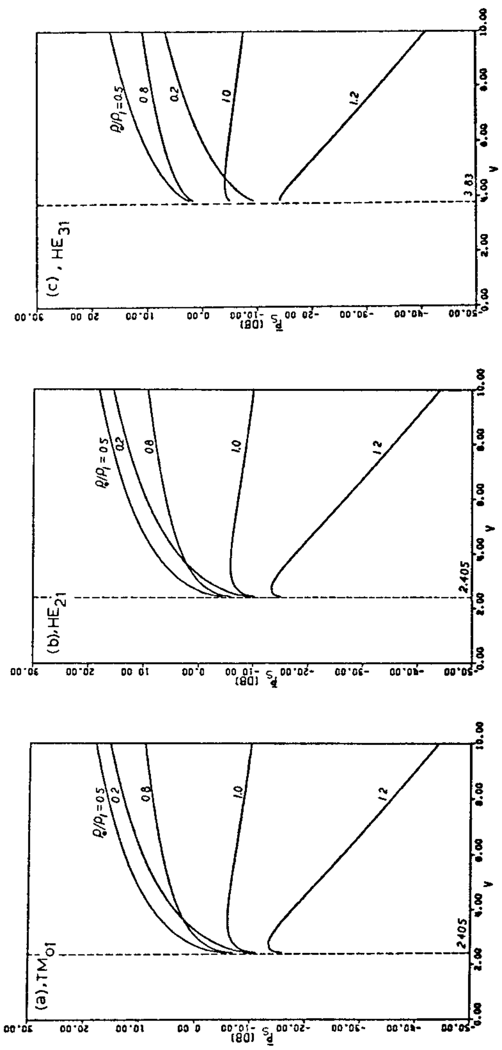

Fig. 4: The normalized scattered modal power $\overline{P}_s$ versus the normalized frequency V. $\varepsilon_{r1} = 2.341$, $\varepsilon_{r2} = 2.250$. (a) TM_{01} (b) HE_{21} (c) HE_{31}

radiation power. The power calculated by this approximate method is sufficiently accurate for waveguides with small dielectric differences between the core and the cladding. It, however, fails to predict the accurate radiation pattern and radiation fields. Comparisons with the approximate results will be made. The spatial distribution of the radiated power, and the radiation power patterns have also been calculated and the results will be presented. The radiation patterns are markedly different from those calculated from the infinite-medium approximation.

It is emphasized that, although we treated the case of a single-mode fiber only, the analytical formulation developed here is capable of handling the general case of an arbitrary incident mode. Furthermore, the analysis permits the study of other cylindrically stratified dielectric waveguides including those used in millimetric communication systems.

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